

Optimal Taxation in Search Equilibrium with Home Production*

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Abstract

The paper develops a two-sector general equilibrium search model where “goods” are produced exclusively in the market and “services” are produced both in the market and within the households. We use the model to examine how unemployment and welfare are affected by labor taxes in general and sectoral tax differentiation in particular. We find that a tax cut on services reduces unemployment whereas a tax cut on goods has no effect. A reform involving tax differentiation, with lower taxes on services, is welfare improving. Numerical calibrations of the model suggest that the welfare gains from tax differentiation are large if the government absorbs a substantial fraction of GDP.

JEL-classification: H21, J22, J41, J64

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1 Introduction

Recent years have seen a revival of interest in “public finance solutions” to unemployment problems.¹ One strand of theoretical and empirical research has explored the effects of general labor taxation. A related but much smaller literature has been concerned with the implications of *tax differentiation* across sectors or workers with different skills. The present paper explores the case for tax differentiation between “goods” and “services”. Goods are exclusively produced in the market whereas services are produced in the market as well as within the households.

Most of the existing literature on taxation and home production has dealt with efficiency aspects, typically under the assumption of competitive labor markets; see for example Boskin (1974), Sandmo (1990) and Kleven *et al* (2000). More recent policy discussions have focused on employment effects of tax reforms involving lower taxes on services that are close substitutes to goods produced within the households. Indeed, several European countries have seen policy initiatives where tax reliefs are introduced on various “household services” such as gardening, catering, cleaning and repair activities. The European Union has recently issued a new directive that extended the range of goods and services that could be subject to reduced tax rates. The motivation for this amendment was explicitly focused on employment objectives (Council directive 1999/85/EC).

The literature on tax policies in economies with home production has only recently addressed issues related to unemployment. Three examples are Fredriksen *et al* (1995), Sørensen (1997) and Kolm (2000). The present paper is most closely related to Kolm’s analysis of tax differentiation in an economy with union bargaining over wages. Like Kolm we consider an economy with two market sectors, one of them producing a commodity that is a perfect substitute to the commodity produced at home. Our model of the labor market is different, however. Kolm’s model ignores job search and is partial equilibrium in the sense that there is no link between bargained wages and general labor market conditions. The present model features endogenous job

¹See for example Sørensen (1997).

search and bargained wages are affected by overall labor market conditions.

Most of the literature on taxes and unemployment has paid little attention to income sources other than labor earnings and unemployment benefits.² A shortcut is to allow for exogenous income or utility components associated with unemployment, such as income from home production or a fixed value of leisure; see, for example, Bovenberg and van der Ploeg (1998), Mortensen (1994) and Mortensen and Pissarides (1998). By contrast, our paper develops a model where the worker's income from home production is endogenously determined. We adopt a search equilibrium framework along the lines of Pissarides (1990/2000) and extend it by incorporating home production and two market sectors.³ Time devoted to home production is here a choice variable for unemployed individuals who allocate their time between job search and home production. The cost of search is thus foregone home production. The unemployed worker's income is the endogenously determined value of home production. The real value of home production depends not only on decisions on time allocation but also on relative prices between goods and services. Tax policies affect relative prices, a fact that has important implications for the effects on labor market outcomes.

Section 2 of the paper presents the basic model of a two-sector economy where services produced in one of the market sectors is a perfect substitute to services produced at home. The model determines real wages, total and sectoral employment as well as the relative price between goods and services. Section 3 turns to the effects of tax policies. We show that a tax cut on services reduces unemployment whereas a tax cut on goods has no effect. We also show, in section 4, that the introduction of sectoral tax differentiation, with lower taxes on services, is welfare improving when the government has an exogenous revenue requirement. Section 5 of the paper presents the results

²Note however the contributions by Phelps where wealth and nonwage income are key elements in the theory of unemployment. See Phelps (1994), Phelps and Zoega (1998), and Hoon and Pelps (1996, 1997).

³Holmlund (2001) presents a one-sector model with home production. Boone and Bovenberg (2000) provide a detailed discussion of taxation in one-sector search models under different assumptions about labor demand conditions.

of numerical calibrations of the model. These exercises suggest that the optimal tax differential between goods and services can be quite large when the government absorbs a substantial fraction of GDP. The welfare gains from optimal tax differentiation are increasing in the government's revenue requirement. Section 6 discusses a number of extensions of the basic model and section 7 concludes.

2 The Model

2.1 The Labor Market

The economy consists of two market sectors. One sector produces goods whereas the other sector produces services. The goods can only be produced in the market, whereas services can either be produced in the market or within the households. We denote goods by G and services by Z .

A worker is either unemployed or employed in one of the market sectors. The labor force is fixed and normalized to unity. Workers and firms are matched according to a constant-returns-to-scale matching function that relates the flow of new hires (H) to the total number of vacancies (v) and the effective number of job searchers. Only unemployed workers search for jobs. In fact, there will be no incentives for on-the-job search in a symmetric equilibrium since there will be no wage differentials across firms or sectors in this case (irrespective of relative tax rates). Let s denote search intensity and u the number of unemployed. The effective number of searchers is then given as su . With the labor force normalized to unity, v and u also represent the vacancy rate and the unemployment rate, respectively. Without loss of generality we take the matching function to be of the Cobb-Douglas form:

$$H = mv^{1-\eta}(su)^\eta \tag{1}$$

where $\eta \in (0, 1)$ and m is a positive scale parameter. We set $m = 1$ in the subsequent theoretical exposition; in the numerical exercises, m will be calibrated along with other parameters of the model.

Workers engage in “undirected” random search for any job, i.e., they do not direct their search towards any particular sector. For a given amount of search effort, the probability of locating an offer from the goods sector, say, depends on the number of vacancies in that sector relative to the total number of vacancies. The transition rate from unemployment to the goods sector thus generally differs from the transition rate to the service sector as the relative supply of vacancies may differ.⁴

The sector-specific vacancy rates are denoted v^j , $j = G, Z$; hence $v = v^G + v^Z$. The unemployed worker’s transition rates into employment can be expressed as $\lambda^G = \gamma sH/su = \gamma s\theta^{1-\eta} = \gamma s\alpha(\theta)$, and $\lambda^Z = (1 - \gamma) sH/su = (1 - \gamma) s\theta^{1-\eta} = (1 - \gamma) s\alpha(\theta)$; $\gamma = v^G/v$ is the fraction of vacancies supplied by the goods sector and $\theta = v/su$ is a measure of overall labor market tightness. The term $s\alpha(\theta)$ can be interpreted as the probability per unit time of getting any job offer, i.e., $\lambda^G + \lambda^Z = s\alpha(\theta)$. The probability per unit time that a firm meets a worker is equal across firms and given by $q(\theta) = H/v = \theta^{-\eta}$. Furthermore, we define labor market tightness for the goods sector as $\theta^G = v^G/su$ and labor market tightness for the service sector as $\theta^Z = v^Z/su$, where $\theta^G + \theta^Z = \theta$.

The steady state unemployment rate and the sector-specific employment rates, n^G and n^Z , are derived from the labor force identity, $n^G + n^Z + u = 1$, and the flow equilibrium conditions. The latter conditions take form $\lambda^G u = \phi n^G$ (the goods sector) and $\lambda^Z u = \phi n^Z$ (the service sector), where ϕ is the exogenous rate at which employed workers are separated from their jobs and enter unemployment. The separation rates are thus assumed to be equal across sectors. Solving for the employment rates and the unemployment rate, we obtain:

$$n^G = \frac{\lambda^G}{\phi + \lambda^G + \lambda^Z} = \frac{\theta^G s\theta^{-\eta}}{\phi + s\theta^{1-\eta}} \quad (2)$$

⁴One interpretation of undirected search is that workers locate employers through a centralized employment agency without knowing in advance the sectoral identity of any vacancy that comes along. We briefly consider “directed” search in Section 6 below. The results are identical to those obtained with undirected search.

$$n^Z = \frac{\lambda^Z}{\phi + \lambda^G + \lambda^Z} = \frac{\theta^Z s \theta^{-\eta}}{\phi + s \theta^{1-\eta}} \quad (3)$$

$$u = \frac{\phi}{\phi + \lambda^G + \lambda^Z} = \frac{\phi}{\phi + s \theta^{1-\eta}} \quad (4)$$

2.2 Workers

Workers are infinitely lived and care about consumption of goods and services. All workers have identical preferences captured by the homothetic instantaneous utility function $v(G, Z)$, where G is the quantity consumed of goods and Z the quantity consumed of services. Market-produced services and home-produced services are perfect substitutes in consumption. When the employed worker acts as a consumer, she takes total income as given by the bargaining agreements on wages and hours.

The worker's time endowment is normalized to unity. The employed worker's time is divided between market work, l^j , and home production, h^j , where index j denotes in which sector the worker is employed, $j = G, Z$. The allocation of time is determined in bargains between the firms and the individual workers. Unemployed workers allocate their time between search, s , and home production, h^u . An employed worker in sector j has the production function $z^j = z(h^j)$; the unemployed worker's production function is analogously given as $z^u = z(h^u)$. These production functions are identical across sectors and labor force states, increasing in time devoted to home production, and strictly concave.

The employed worker's instantaneous income is given by $I^j = w^j l^j + R + \pi + P^Z z(h^j)$, where w^j is the hourly wage and l^j the number of hours allocated to market production. R is a lump sum transfer received from the government, π the share of profits received as dividends, and P^Z the price of services. The aggregate profits generated in the economy are distributed equally across the population. The unemployed worker's instantaneous income is analogously given by $I^u = R + \pi + P^Z z(h^u)$. We thus ignore unemployment benefits; the implications of accounting for benefits are briefly discussed in section 6 below.

We can now define the value functions. Let U , E^G and E^Z denote the expected present values of unemployment and employment in the two market sectors, respectively. The value functions for worker i can then be written as follows:

$$\begin{aligned} rU_i &= \frac{P^Z z_i^u + R + \pi}{P} + \lambda^G (E^G - U_i) + \lambda^Z (E^Z - U_i) \\ &= \frac{P^Z z_i^u + R + \pi}{P} + s_i \alpha(\theta) [\gamma E^G + (1 - \gamma) E^Z - U_i] \end{aligned} \quad (5)$$

$$rE_i^G = \frac{w_i^G l_i^G + P^Z z_i^G + R + \pi}{P} + \phi(U - E_i^G) \quad (6)$$

$$rE_i^Z = \frac{w_i^Z l_i^Z + P^Z z_i^Z + R + \pi}{P} + \phi(U - E_i^Z) \quad (7)$$

where $P = P(P^G, P^Z)$ is the general cost-of living index implied by homothetic preferences. E^j is the value of employment in an arbitrary firm in sector j .

The unemployed worker chooses search intensity, s_i , in order to maximize the value of unemployment, rU_i .⁵ The first-order condition for an interior solution takes the form:

$$\frac{P^Z z_i'(h_i^u)}{P} = \alpha(\theta) [\gamma E^G + (1 - \gamma) E^Z - U_i] \quad (8)$$

The left-hand side is the marginal cost of increasing search, i.e., the real value of foregone home production. The higher the relative price of services, the higher the value of foregone home production and thus the higher the marginal cost of search. The right-hand side is the expected return from an increase in search effort. The tighter the labor market, the higher the return to search. It follows immediately that the unemployed worker's search effort is decreasing in the relative price of services and increasing in labor market tightness.

⁵One can think of the unemployed worker's behavior as if she first acts as a producer by selling untaxed services to the market in order to maximize profits. In the second stage she acts as a consumer, choosing optimally between goods and services.

2.3 Firm Behavior and Wage Bargaining

Let J^j and V^j represent the expected present values of an occupied job and a vacant job, respectively. The marginal product of a worker is constant and denoted y . The (nominal) cost of holding a vacancy open is $P^j \kappa y$. One interpretation of this specification of the vacancy cost is that the firm allocates its workforce optimally between production and recruitment activities. The cost of hiring is its alternative cost, i.e., the value of the marginal product of labor.⁶ There is a proportional payroll tax rate pertaining to sector j that is denoted t^j . (The results are essentially identical with value added taxes, as is briefly discussed in Section 6.) The arbitrage equations are then given as follows:

$$rJ_i^j = \frac{P^j}{P} l_i^j y - \frac{w_i^j (1 + t^j) l_i^j}{P} + \phi(V^j - J_i^j) \quad j = G, Z \quad (9)$$

$$rV^j = -\frac{P^j \kappa y}{P} + q(\theta)(J^j - V^j) \quad j = G, Z \quad (10)$$

With free entry of vacancies we can impose $V^j = 0$ and use (9) and (10) to derive a “feasible” real producer wage (also referred to as the “zero-profit condition”):

$$\frac{w^j (1 + t^j) l^j}{P^j} = y \left[l^j - \frac{\kappa (r + \phi)}{q(\theta)} \right] \quad j = G, Z \quad (11)$$

The feasible wage depends on total labor market tightness and hours of work. The higher is tightness, the lower the feasible wage since expected recruitment costs are higher. A rise in working time raises the feasible wage, which can be thought of as a productivity effect of longer hours. If there is a rise in work-hours, the firm can transfer some workers from recruitment activities to production while keeping its total labor force constant. This raises output per employee, implying a higher feasible real wage.

The firm and the worker bargain over the hourly wage and the number of work-hours. Prices are taken as given. The Nash bargaining problem for

⁶See Holmlund (2001) for further discussion of and motivation for this specification of vacancy costs in a model with endogenous work-hours.

a particular firm-worker pair i is given by:

$$\max_{w_i, l_i} \Omega_i^j(w_i, l_i) = [E_i^j(w_i, l_i) - U]^\beta [J_i^j(w_i, l_i) - V^j]^{1-\beta} \quad j = G, Z$$

The first-order conditions derived by maximizing the Nash product with respect to w_i^j , and l_i^j , can be written as

$$E^j - U = \frac{\beta}{1-\beta} \frac{J^j}{1+t^j} \quad j = G, Z \quad (12)$$

$$E^j - U = \frac{\beta}{1-\beta} \left[\frac{P^Z z'(h^j) - w^j}{P^j y - w^j (1+t^j)} \right] J^j \quad j = G, Z \quad (13)$$

where we have imposed symmetry and the free entry condition, $V^j = 0$. Work-hours are obtained by dividing eqs. (12) and (13) for $j = G, Z$, respectively. We then obtain:

$$P^Z z'(h^G) = \frac{P^G y}{1+t^G} \quad (14)$$

$$P^Z z'(h^Z) = \frac{P^Z y}{1+t^Z} \quad (15)$$

Eqs. (14) and (15) state that the value of the marginal product in home production (the left-hand sides) equals the tax adjusted value of the marginal product in market work (the right-hand sides). Note that an increase in the relative price of services increases home production (reduces work-hours) in the goods sector. In the service sector, however, relative price changes have no effect on time allocation since the effects on the value of home production and the value of market production offset each other.

Bargained wages in the two sectors as functions of labor market tightness can be solved from (12), using also eqs. (5) – (7), (9), (10) and imposing the free entry condition $V = 0$. The equations for bargained real producer wages can be written as follows:

$$\frac{w^G (1 + t^G) l^G}{P^G} = (1 - \beta) \left[(z^u - z^G) \left(\frac{P^Z}{P^G} \right) (1 + t^G) \right] + \beta y (l^G + \kappa s \theta^G) + \beta y \kappa s \theta^Z \Delta^{-1} \quad (16)$$

$$\frac{w^Z (1 + t^Z) l^Z}{P^Z} = (1 - \beta) [(z^u - z^Z) (1 + t^Z)] + \beta y (l^Z + \kappa s \theta^Z) + \beta y \kappa s \theta^G \Delta \quad (17)$$

where Δ is defined as

$$\Delta \equiv \frac{P^G}{P^Z} \frac{(1 + t^Z)}{(1 + t^G)} \quad (18)$$

and referred to as the wedge.

Bargained wages in a particular sector depend on labor market tightness in both sectors, an implication of the fact that the unemployed worker searches over both sectors. Treating the relative price of services as fixed for a moment, the first line on the right-hand side of eq. (16) captures the impact of conditions in the G -sector on wages in that sector; analogously, the first line of eq. (17) captures own-sector effects on Z -wages. Note also that wages depend on home production; the larger home production when unemployed relative to home production when employed, the higher are bargained real wages.

There is a slight but important asymmetry between the two wage equations. Consider the first lines of eqs. (16) and (17). In both equations, payroll tax rates interact with the productivity differential $z^u - z^j$. The tax rates also interact with the relative price of services, P^Z/P^G , but only in the goods sector. The first line of (16) includes a relative price term that captures the value of home production relative to the value of market output in the goods sector. The higher the value of home production relative to the value of market production, the higher the wage pressure in that sector. The relative price does not appear in the service sector by the assumption of perfect substitutability between services produced in the market and in the household. In the service sector, a lower tax rate reduces labor costs at given tightness and given wedge, assuming $z^u > z^Z$. An analogous argument does not necessarily hold for tax changes in the goods sector since relative price

changes may offset the “direct” effect of a tax change. It turns out that this asymmetry has important implications for how sectoral tax differentiation affects labor market outcomes.

2.4 Equilibrium

The general equilibrium of the model can be solved in a convenient recursive fashion. By first combining the zero-profit conditions and the wage equations we can determine total tightness and the relative price. Knowing tightness and the relative price we can determine search effort and work-hours, and thus also household production among employed and unemployed workers. The unemployment rate is obtained once we know total tightness and search effort. The sectoral allocation of tightness is finally determined by means of an equation that equates aggregate demand and aggregate supply of goods and services.

We begin with the determination of total tightness and the relative price. Use the zero-profit conditions and the wage equations, i.e., eqs. (11), (16) and (17), and obtain equations of the form:

$$\kappa(r + \phi)\theta^\eta = (1 - \beta) \left[l^G - \frac{1}{y\Delta} (z^u - z^G) (1 + t^Z) \right] - \beta\kappa s \left(\theta^G + \frac{\theta^Z}{\Delta} \right) \quad (19)$$

$$\kappa(r + \phi)\theta^\eta = (1 - \beta) \left[l^Z - \frac{1}{y} (z^u - z^Z) (1 + t^Z) \right] - \beta\kappa s \Delta \left(\theta^G + \frac{\theta^Z}{\Delta} \right) \quad (20)$$

where we have used the definition of the wedge in (18) to substitute out $(P^Z/P^G)(1 + t^G)$ from eq. (16). Firms will enter into the two sectors until the expected discounted profits are equal to the expected vacancy costs. The expected time it takes to fill a vacancy, $1/q(\theta) = \theta^\eta$, is equal across firms in the two sectors, although vacancy costs per period may differ across sectors as output prices may differ.

Note that the derivatives of the right-hand sides of (19) and (20) with respect to l^j are zero, an implication of the fact that work-hours are optimally

determined in the bargains. The right-hand sides are also invariant to derivative changes of s when search effort is optimally determined by the worker, recognizing also the free entry condition for vacancies and the sharing rule for wages.⁷

In order to characterize the equilibrium it is useful to invoke the following lemma:

Lemma 1: *The equilibrium allocation of time involves $h^u > h^j$ and hence $l^j > s$ as well as $z^u > z^j$, $j = G, Z$.*

Proof See Appendix A. ■

The model has thus the empirically plausible implication that unemployed workers spend more time in home production than employed workers do. As will become clear in the subsequent analysis, this property has also implications for how taxes affect labor market outcomes.

Eqs. (19) and (20) include three endogenous variables, i.e., θ^G , θ^Z and Δ (once l^j and s are substituted out by means of the relevant first-order conditions). We can achieve considerable simplification by solving for the wedge and obtain:

$$\Delta = \frac{(1 - \beta) \left[l^Z - \frac{1}{y} (z^u - z^Z) (1 + t^Z) \right] - \kappa (r + \phi) \theta^\eta}{(1 - \beta) \left[l^G - \frac{1}{y\Delta} (z^u - z^G) (1 + t^Z) \right] - \kappa (r + \phi) \theta^\eta} \quad (21)$$

where the derivatives of the right-hand side with respect to l^j and s are zero. Inspection of (21) reveals that $\Delta = 1$ is a solution to the equation. It can be shown that this is also the unique solution:

Lemma 2: *The unique equilibrium of the model entails $\Delta = 1$, i.e., $P^Z/P^G = (1 + t^Z)/(1 + t^G)$.*

Proof See Appendix B. ■

⁷Use (8), (10) and (12) and obtain:

$$z'(h^u) = \left(\Delta \theta^G + \theta^Z \right) \left(\frac{\beta}{1 - \beta} \right) \left(\frac{\kappa y}{1 + t^Z} \right).$$

The solution has an important property, namely that the relative price is fixed by the relative tax ratio. The relative price is thus independent of consumers' preferences for goods and services. A tax cut on services increases the supply of services relative to the supply of goods, which in turn induces a decline in the price of services relative to the price of goods.

With $\Delta = 1$ we can use eqs. (14) and (15) to determine equilibrium hours of work:

$$z'(h^G) = z'(h^Z) = \frac{y}{1+t^Z} \quad (22)$$

Work-hours are thus equal across sectors, i.e., $l^G = l^Z = l$, implying also $z^G = z^Z = z^e$. Search intensity by the unemployed worker, as a function of total labor market tightness, is obtained by invoking $\Delta = 1$ together with eqs. (8) and (12):

$$z'(h^u) = \theta \left(\frac{\beta}{1-\beta} \right) \left(\frac{\kappa y}{1+t^Z} \right) \quad (23)$$

By imposing $\Delta = 1$ in eq. (19) and recognizing (22) and (23) we get an equation that determines total tightness:

$$\kappa(r + \phi)\theta^\eta = (1 - \beta) \left[l - \frac{1}{y}(z^u - z^e)(1 + t^Z) \right] - \beta \kappa s \theta \quad (24)$$

where the right-hand side is invariant to derivative changes of l and s , an envelope property already alluded to. With total tightness and search effort determined we obtain the unemployment rate from eq. (4) and real wages from eqs. (16) and (17). Note also that the equilibrium with $\Delta = 1$ involves equal wages across sectors, i.e., $w^G = w^Z = w$.

The equilibrium outcomes described so far are entirely supply determined, i.e., they do not depend on shifts in consumer preferences for goods and services. However, to determine sectoral employment levels we need to explicitly consider the demand side. For completeness we sketch also this derivation.

With homothetic preferences we have the aggregate demand function for the two goods given from the first-order condition for the individual consumer's optimal mix of commodities, i.e., $v_G(G, Z)/v_Z(G, Z) = P^G/P^Z$,

in conjunction with the aggregate budget constraint. The relative price is obtained by equating demand and supply of commodities. The aggregate supply of goods is given by $Y^G = yn^Gl - v^G\kappa y$, whereas the aggregate supply of services is given by the supply of market produced services and the aggregate volume of home production, i.e., $Y^Z = yn^Zl - v^Z\kappa y + (1 - u)z^e + uz^u$. Equate aggregate demand and aggregate supply and use the fact that preferences are homothetic to obtain an equation for the relative price: $v_G(Y^G/Y^Z, 1)/v_Z(Y^G/Y^Z, 1) = P^G/P^Z$. The relative price P^G/P^Z is declining in the relative supply of goods, Y^G/Y^Z , due to normality, i.e., $\partial(P^G/P^Z)/\partial(Y^G/Y^Z) < 0$. Next we use eqs. (2) – (4), (18) and $\Delta = 1$ to substitute out the relative price and obtain a relationship between total tightness, θ , and relative tightness, i.e., θ^Z/θ^G :

$$\frac{1 + t^G}{1 + t^Z} = \frac{v_G(Y^G/Y^Z, 1)}{v_Z(Y^G/Y^Z, 1)} \quad (25)$$

where

$$\frac{Y^G}{Y^Z} = \frac{y(l - \kappa\phi\theta^\eta)}{y\frac{\theta^Z}{\theta^G}(l - \kappa\phi\theta^\eta) + \left(1 + \frac{\theta^Z}{\theta^G}\right)(z^e + \phi z^u\theta^{\eta-1}s^{-1})} \quad (26)$$

With total tightness and time allocation already determined, we obtain relative tightness from (25) and (26). Relative sector employment is obtained by noting that $n^Z/n^G = \theta^Z/\theta^G$ is implied by the flow equilibrium conditions. This completes the analysis of the equilibrium of the model.⁸

3 Tax Policy

We now examine how tax policies affect labor market outcomes. The government's budget restriction is given by:

$$t^Z w l n^Z + t^G w l n^G = R \quad (27)$$

⁸We have ignored the government's budget restriction so far, assuming that the budget can always be balanced by adjustment of the lump sum subsidy to the individuals.

where R is the lump sum transfer to the individuals. A cut in t^Z can be financed either by an adjustment in t^G or by an adjustment of R . We can derive the following results:

Proposition 1 (i) *A reduction in t^Z followed by adjustments in t^G or R increases total tightness, reduces unemployment and increases hours of work and search intensity.* (ii) *A change in t^G followed by an adjustment in R has no effect on total tightness, unemployment, hours of work and search intensity.*

Proof Differentiate eq. (24) implicitly to derive the effect on tightness, recognizing Lemma 1 and the envelope property that (24) is invariant to derivative changes in search and work-hours. Use eqs. (22) and (23) to derive the effects on work-hours and search intensity and eq. (4) to obtain the effect on unemployment. Note that t^G and R do not appear in any of the relevant equations and hence cannot affect any of the variables of interest. ■

The intuition for these results are as follows. Consider a cut in the service sector payroll tax rate, t^Z . The immediate effect is a decline in producer costs among firms producing services. This initiates entry of new vacancies into the service sector, thereby increasing the value of unemployment. Bargained wages therefore rise in both sectors. Higher producer costs in the goods sector drive firms out of that sector and into the service sector. This sectoral reallocation of firms increases the market production of services and reduces the market production of goods, thereby reducing the price of services relative to the price of goods. The relative price adjustments will restore the equilibrium since the increase in the price of goods eventually makes it profitable to produce goods and the reduced price on services eventually eliminate the profitability of entering the service sector. The new equilibrium is associated with lower producer costs in both sectors and lower unemployment.

Why does not the rise in bargained wages completely offset the decline in producer costs? The reason is the presence of untaxed home production, and in particular the fact that home production during unemployment exceeds home production during employment, i.e., $z^u > z^e$. This is immediately clear from eq. (24); taxes on services would not matter in the absence of home

production. With untaxed home production, however, a policy that raises workers' take-home pay in the market sector makes it more valuable to be employed relative to being unemployed. A tax cut therefore increases the attractiveness of employment relative to unemployment. Stated differently, a tax cut makes it less attractive to engage in tax avoidance by working in the household sector.

This result can be compared to a standard feature of equilibrium models of unemployment, namely that taxes are neutral with respect to unemployment as long as unemployment benefits are indexed to real take-home wages (see, for example, Pissarides, 1998). With a fixed replacement rate in unemployment insurance, and absent home production, any tax cut is typically completely offset by a rise in wages; the indexation of benefits to wages introduces additional upward pressure on wages as taxes are reduced. Our model would reproduce the standard result if home production were ignored and the sectors were completely symmetric (as we have assumed).⁹

It is useful to glance at the equations for bargained wages – eqs. (16) and (17) – to get some further feel for the mechanisms involved. Consider first how a cut in t^Z affects the service sector. A lowering of t^Z induces a downward shift of the wage-setting schedule; i.e., a decline in real producer wages given tightness. This allows an expansion of market output and employment in the service sector. Note that the price of home produced commodities relative to the price of market production is fixed in the service sector, an implication of perfect substitutability between household-produced services market-produced services.

Now consider how a cut in t^Z affects bargained wages in the goods sector. A reduction of t^Z produces a downward shift of the wage-setting schedule also in the goods sector, but the reason here is the relative price adjustment. The tax cut on services induces a decline in the value of services (home

⁹If the sectors had not been symmetric, it would in general be possible to influence total tightness and unemployment by changes in relative tax pressure even absent home production. This is analogous to the effects of sectoral tax policies in non-symmetric two-sector models of union bargaining discussed in Kolm (1998) and Holmlund and Kolm (2000).

production) relative to the market value of goods, i.e., a decline in P^Z/P^G .

As work-hours are determined in the bargains by the tax adjusted value of the marginal product in market work, hours in the service sector increase with a reduction in t^Z as market work becomes less taxed relative to home work. Hours increase also in the goods sector with a reduction in t^Z as the relative price P^Z/P^G falls, which raises the value of market work relative to home work in the sector. Search intensity increases with a reduction in t^Z both because t^Z is directly reduced and because total tightness increases. Both effects increase the returns to search.

Changes in t^G do not affect output, employment and hours in our model. Budget balance can thus be achieved by adjustment of t^G without any repercussions on the labor market outcomes of main interest. Consider again the equations for bargained wages. Had the relative price been fixed, a cut in t^G would lower real producer costs in the goods sector, thereby raising employment. But the relative price is not fixed; it is in fact highly responsive to tax changes. A cut in t^G causes an equiproportionate increase in P^Z/P^G as the supply of goods increases relative to the supply of services. This relative price adjustment completely offsets the tax reduction. This adjustment in the value of home production is analogous to the adjustment of benefit levels that takes place when the replacement rate is fixed.

Moreover, changes in t^G will have no impact on work-hours and search intensity. Hours in the service sector are not affected as t^G has no impact on the tax adjusted value of the marginal product in market work. Hours in the goods sector are directly affected by changes in t^G but this effect is counteracted by adjustments in the relative price, leaving work-hours in the goods sector unaltered as well. Search intensity is unaffected by adjustment in t^G as total tightness is unaffected and so is the tax adjusted value of the marginal product in market work in the two sectors. Hence, as the pay-off to search is unaffected by changes in t^G , so is search intensity.

4 Welfare

We have seen that a tax cut on services unambiguously reduces unemployment. Does such a reform also represent a welfare improvement? Consider a social welfare function of the utilitarian form:

$$W = n^G r E^G + n^Z r E^Z + u r U \quad (28)$$

Substitute the expressions for the value functions given by eqs. (5) – (7) into eq. (28), impose the flow equilibrium conditions given by eqs. (2) – (4) and the government's budget restriction (27), and substitute the expression for aggregate profits into eq. (28). Finally take the limit of the resulting expression as the discount rate approaches zero, i.e., $r \rightarrow 0$. By ignoring discounting we can compare different steady states without having to consider the adjustment process. This yields the following expression for social welfare:

$$W = \frac{P^G Y^G + P^Z Y^Z}{P(P^G, P^Z)} \quad (29)$$

where $Y^G = n^G (l - \kappa \phi \theta^\eta) y$ is aggregate consumption of goods, and $Y^Z = n^Z (l - \kappa \phi \theta^\eta) y + (1 - u) z^e + u z^u$ is the aggregate consumption of services which consists of market produced services as well as home produced services. The social welfare measure is thus simply given by real aggregate consumption. By dividing the numerator and the denominator by P^Z , and using the fact that the price level is linearly homogenous in the sector prices, we obtain a social welfare measure of the following form:

$$W = \frac{(P^G/P^Z) Y^G + Y^Z}{P(P^G/P^Z, 1)} \quad (30)$$

Does a tax reform that involves a switch from uniform to differentiated taxation represent a welfare improvement? The following proposition summarizes the results:

Proposition 2 *Consider an initial situation with uniform taxation, i.e., $t^G = t^Z = t \geq 0$. (i) Social welfare is invariant to tax differentiation provided*

that the government has no revenue requirement, i.e., $t = 0$, and provided that $\beta = \eta$ holds. (ii) Social welfare is increased by tax differentiation involving $t^G > t^Z$ provided that the initial tax rates are positive, i.e., $t > 0$, and $\beta \geq \eta$.

Proof Differentiate (30) with respect to t^Z while adjusting t^G so as to recognize the government's budget restriction, $t^G = t^G(t^Z)$. Evaluate at $t^Z = t^G = t$ and obtain:

$$\text{sign} \left(\frac{dW}{dt^Z} \right)_{t^G(t^Z)} = \text{sign} \left\{ \Omega_1 (\beta - \eta) \frac{\partial \theta}{\partial t^Z} + t \left[\Omega_2 \frac{\partial \theta}{\partial t^Z} + \Omega_3 \frac{\partial s}{\partial t^Z} + \Omega_4 \frac{\partial l}{\partial t^Z} \right] \right\}$$

where $\Omega_i > 0$, $i = 1, \dots, 4$, $\partial \theta / \partial t^Z < 0$, $\partial s / \partial t^Z < 0$ and $\partial l / \partial t^Z < 0$; see Appendix C for definitions of Ω_i and other details. Thus: (i) $dW/dt^Z = 0$ if $\beta = \eta$ and $t = 0$; (ii) $dW/dt^Z < 0$ if $\beta \geq \eta$ and $t > 0$. ■

The first part of the proposition is a restatement of the so-called Hosios condition: the policy-free equilibrium, i.e., $t = 0$, is constrained efficient provided that the elasticity of matching with respect to the effective number of searchers is equal to the power of the worker in the Nash bargain, i.e., $\beta = \eta$. There is then no reason to use sectoral tax differentiation so as to remove inefficiencies caused by search externalities. The second part of the proposition states that tax differentiation is always welfare improving when the government has a revenue requirement provided that $\beta \geq \eta$ holds; note that $\beta \geq \eta$ is a sufficiency condition. If a policy-free equilibrium involved $\beta > \eta$, it would imply that tightness would be too low and unemployment too high.

The incentives for tax differentiation when $t > 0$ arise from three “fiscal” externalities associated with tightness, search and work-hours; cf. the expressions within the squared brackets above. A cut in t^Z accompanied by a rise in t^G is welfare improving by increasing tightness, increasing search and increasing work-hours. These changes raise total man-hours and thereby the tax base. This in turn allows a rise in total private consumption without any offsetting decline in government revenues.

In conclusion, we have derived sufficient conditions under which tax differentiation would be welfare improving. It follows immediately that a uniform tax structure cannot be optimal; there will always exist a better alternative where taxes on services are lower than taxes on goods. As we will see, the welfare gains from a switch from a uniform to a differentiated tax system may well be substantial.

5 Numerical Results

We now turn to numerical calibrations of the model so as to get some feel for the magnitude of the optimal tax differentiation and the associated welfare gains. Preferences for goods and services are represented by a Cobb-Douglas utility function. To check how sensitive the results are to alternative assumptions concerning preferences we also consider utility functions of the CES variety.

With Cobb-Douglas preferences we have $v(G, Z) = G^\sigma Z^{1-\sigma}$, where we set $\sigma = 0.5$. The matching function is given by $H = mv^{1-\eta}(su)^\eta$. We also assume that the worker's share of the total match surplus equals the elasticity of matching with respect to unemployment, i.e., $\beta = \eta$; this is the Hosios-condition already referred to. We set $\beta = \eta = 0.5$. The home production functions are identical across sectors and labor market states and of the form:

$$z^j = ay (h^j)^b \quad j = G, Z \quad (31)$$

and analogously for $z^u = z(h^u)$. The production functions are strictly concave so $b < 1$. The assumption that productivity in home production rises along with productivity in market production has the realistic implication that unemployment is constant along a balanced growth path, i.e., unemployment is independent of the level of productivity.¹⁰ The unemployed worker's time in home production is obtained by using eqs. (22) – (24). The day

¹⁰It also follows that the worker's time allocation is independent of the level of productivity. To see that productivity does not affect time allocation and unemployment, invoke (22) to obtain $\partial l^j / \partial y = 0$ and use (23), (24) and (4) to obtain $\partial s / \partial y = \partial l^j / \partial y = \partial u / \partial y = 0$.

is taken as time unit, y is normalized to 100 and the separation rate, equal across sectors, is given as $\phi = 0.25/365$. The parameters κ , a , b and m were chosen so as to obtain “reasonable” values of unemployment and the elasticity of hours with respect to tax rates, i.e., $\xi_t^l \equiv -\partial \ln l^j / \partial \ln(1+t)$. The implied elasticity of ξ_t^l for a uniform tax rate is 0.3 when the government absorbs 25 percent of GDP.¹¹ Note however that our functional form assumption implies that the elasticity is increasing in the tax rate; we have $\xi_t^l = -(h/l)(1-b)^{-1}$ where h/l increases in the tax rate. We set the real interest rate to zero so that we only need to compare steady states.

A utilitarian welfare function can, by the linear homogeneity of the utility function, be represented by real aggregate consumption as in eq. (29) above. We assume that the government has a real revenue requirement, $\bar{R} \equiv R/P$, and returns tax revenues as lump-sum real transfers to each individual in the economy. The two policy instruments are the tax rates, t^G and t^Z .

The results of the simulations with Cobb-Douglas utility are presented in Table 1. Consider first the experiment with uniform taxation. A rising revenue requirement is associated with steeply rising tax rates. The unemployment rate increases from 6.2 percent to 8.2 percent as payroll tax rates rise from 14 to 81 percent. A “Laffer effect” kicks in at $\bar{R} = 30$ with uniform taxation. It is then possible to reduce t^Z without any compensating increase in t^G .

Consider next what happens when the government chooses the two tax rates optimally. The magnitude of the optimal tax differential is substantial when the government absorbs a large fraction of GDP. For example, we have $t^G = 0.75$ and $t^Z = 0.40$ when the revenue requirement amounts to approximately 40 percent of GDP. There is only a slight increase in unemployment associated with rising revenue requirements when taxes are optimally differentiated.

¹¹The survey by Blundell and MaCurdy (1999) reports estimates of labor supply elasticities around 0.10 on average for males and around 0.7 on average for females. These elasticities are typically estimated under the assumption that work-hours are at the worker’s discretion.

Table 1. The Effects of Tax Differentiation, Cobb-Douglas utility.

Parameters: $\beta = \eta = .5$, $y = 100$, $\kappa = .674$, $a = .5, b = .6$, $m = .01$, $r = 0$, $\phi = .25/365$, $\sigma = 0.5$.

\bar{R}	$\frac{\bar{R}}{GDP_{unif}}$	t	t^G	t^Z	u	u	n^G	n^G	n^Z	n^Z	ΔW
		unif	opt	opt	unif	opt	unif	opt	unif	opt	%
10	.12	.14	.16	.11	.062	.061	.53	.51	.41	.42	.0
20	.25	.34	.39	.25	.065	.064	.54	.51	.39	.43	.3
25	.33	.49	.57	.33	.069	.065	.56	.50	.37	.44	.8
26	.35	.53	.61	.34	.070	.065	.56	.50	.37	.44	1.0
27	.37	.58	.65	.36	.071	.066	.57	.50	.36	.44	1.2
28	.39	.63	.70	.38	.073	.066	.58	.49	.35	.44	1.7
29	.41	.70	.75	.40	.076	.066	.58	.49	.34	.45	2.3
30	.45	.81	.81	.42	.082	.067	.60	.48	.32	.45	3.7

The last column shows welfare changes, measured in percent of total consumption, of moving from a uniform to an optimally differentiated tax system. The welfare gain amounts to 1 percent when the government absorbs 35 percent of GDP; the corresponding gain is 3.7 percent when 45 percent of GDP is absorbed. Taken at face values, these numbers suggest substantial welfare gains arising from optimal tax differentiation.

How sensitive are these results with respect to the particular utility function? We have also considered a utility function of the constant elasticity of substitution variety, i.e.,

$$v(G, Z) = [\delta G^{-\chi} + (1 - \delta) Z^{-\chi}]^{\frac{-1}{\chi}} \quad (32)$$

for $-1 \leq \chi < \infty$. The elasticity of substitution is given by $\epsilon_s = 1/(1 + \chi)$. We examine two cases with “low” and “high” elasticity of substitution: $\epsilon_s = 1/3$ and $\epsilon_s = 2$. The other parameters are left intact. The results are given in Table 2 and Table 3.

Table 2. The Effects of Tax Differentiation, CES utility with $\epsilon_s = 1/3$.
The other parameters are as in Table 1.

\bar{R}	$\frac{\bar{R}}{GDP_{unif}}$	t	t^G	t^Z	u	u	n^G	n^G	n^Z	n^Z	ΔW
		unif	opt	opt	unif	opt	unif	opt	unif	opt	%
10	.12	.14	.18	.08	.062	.061	.53	.51	.41	.42	.1
20	.25	.34	.44	.17	.065	.062	.54	.51	.40	.43	.5
25	.33	.50	.62	.21	.069	.063	.56	.51	.37	.43	1.2
26	.35	.53	.67	.22	.070	.063	.56	.51	.37	.43	1.5
27	.37	.58	.71	.23	.071	.063	.57	.50	.36	.43	1.9
28	.39	.63	.75	.24	.073	.063	.58	.50	.35	.43	2.4
29	.41	.70	.80	.24	.076	.063	.58	.50	.34	.43	3.2
30	.45	.81	.85	.25	.082	.064	.60	.50	.32	.44	4.7

Table 3. The Effects of Tax Differentiation, CES utility with $\epsilon_s = 2$.
The other parameters are as in Table 1.

\bar{R}	$\frac{\bar{R}}{GDP_{unif}}$	t	t^G	t^Z	u	u	n^G	n^G	n^Z	n^Z	ΔW
		unif	opt	opt	unif	opt	unif	opt	unif	opt	%
10	.12	.14	.15	.12	.062	.062	.53	.51	.41	.42	.0
20	.25	.34	.37	.28	.065	.064	.54	.50	.39	.43	.2
25	.33	.49	.54	.39	.066	.066	.56	.49	.37	.44	.5
26	.35	.53	.58	.41	.070	.067	.56	.49	.37	.44	.6
27	.37	.58	.63	.44	.071	.067	.57	.48	.36	.45	.8
28	.39	.63	.68	.46	.073	.068	.58	.48	.35	.45	1.2
29	.41	.70	.73	.49	.076	.069	.58	.48	.34	.46	1.7
30	.45	.81	.79	.52	.082	.070	.60	.47	.32	.46	2.9

The basic message from the CES experiments is that the optimal tax differential is considerably larger when the lower value of the elasticity of substitution applies. Moreover, the welfare gain from tax differentiation is

much higher in this case. The reason why the elasticity of substitution matters is that it affects consumption choices in response to tax induced changes in relative prices. If the elasticity is large, a given tax differentiation induces large changes in consumer demand towards services and away from goods. If the elasticity is small, a given tax differentiation produces only modest changes of consumption decisions. In the extreme case, as the CES function approaches the Leontief function with $\varepsilon_s \rightarrow 0$, there will be no changes in consumption decisions. The less substitutable goods and services are, the more scope for tax differentiation as the associated distortions of consumption decisions are less pronounced. Indeed, with $\varepsilon_s \rightarrow 0$ we find that $t^Z \rightarrow 0$. Taxes should in this case be exclusively levied on the goods sector.

6 Extensions

6.1 Directed Search

We have assumed undirected random search, i.e., workers do not direct their search towards any particular sector. It is arguably more realistic to consider directed search where workers choose which sector to search in; some unemployed workers apply for jobs in the goods sector and the rest in the service sector. We sketch a version of the model with directed search and show that equilibrium outcomes are independent of whether search is directed or undirected.

Suppose that there are identical sector-specific matching functions of the form

$$H^j = (v^j)^{1-\eta} (s^j u^j)^\eta \quad (33)$$

where u^j is the number of workers that allocate search to sector j . From this matching function we get the expressions for transitions rates from unemployment, $\alpha(\theta^j) = (\theta^j)^{1-\eta}$ and the rate at which firms fill vacancies, $q(\theta^j) = (\theta^j)^{-\eta}$. Notice that these rates depend on the number of effective searchers in sector j .

The value functions for a worker can then be written as:

$$rU^j = \frac{P^Z z^{uj} + R + \pi}{P} + s^j \alpha(\theta^j)(E^j - U^j) \quad (34)$$

$$rE^j = \frac{w^j l^j + P^Z z^{ej} + R + \pi}{P} + \phi(U^j - E^j) \quad (35)$$

for $j = G, Z$. Home production in sector j is denoted z^{uj} if the worker is unemployed and as z^{ej} if she is employed. A natural equilibrium condition in this setting is the indifference condition $rU^G = rU^Z$. Workers choose which sector to search in on the basis of a comparison between the present values of search. The pools of searchers adjust so that indifference holds in equilibrium.

Given these assumptions, it is straightforward to derive equations for search, wages and tightness. The arbitrage equations for firms have the same basic structure as with undirected search although it is sector-specific tightness that matters. The free entry condition takes the form:

$$\frac{w^j (1 + t^j) l^j}{P^j} = y \left[l^j - \frac{\kappa (r + \phi)}{q(\theta^j)} \right] \quad (36)$$

and the equations for bargained real producer wages can be written as

$$\frac{w^j (1 + t^j) l^j}{P^j} = (1 - \beta) \left[(z^{uj} - z^{ej}) \left(\frac{P^Z}{P^j} \right) (1 + t^j) \right] + \beta y (l^j + \kappa s^j \theta^j) \quad (37)$$

which corresponds to the first lines of the wage equations with undirected search, i.e., eqs. (16) and (17). Only sector-specific labor market conditions matter with directed search. The free entry conditions and the wage equations imply equations for tightness of the form:

$$\kappa (r + \phi) (\theta^G)^\eta = (1 - \beta) \left[l^G - \frac{1}{y\Delta} (z^{uG} - z^{eG}) (1 + t^Z) \right] - \beta \kappa s^G \theta^G \quad (38)$$

$$\kappa (r + \phi) (\theta^Z)^\eta = (1 - \beta) \left[l^Z - \frac{1}{y} (z^{uZ} - z^{eZ}) (1 + t^Z) \right] - \beta \kappa s^Z \theta^Z \quad (39)$$

where Δ is the wedge as defined above.

By using the first-order conditions for optimal search and optimal hours for the service sector we note that (39) determines θ^Z . Using the first-order conditions we then also get l^Z and s^Z , as well as z^{eZ} and z^{uZ} . To determine the wedge, invoke the indifference condition $rU^G = rU^Z$ in conjunction with the sharing rule for wages. The result can be written as:

$$z^{uZ} + s^Z \theta^Z \left(\frac{\beta}{1 - \beta} \right) \left(\frac{\kappa y}{1 + t^Z} \right) = z^{uG} + s^G \theta^G \left(\frac{\beta}{1 - \beta} \right) \left(\frac{\kappa y}{1 + t^Z} \right) \Delta \quad (40)$$

As with undirected search, $\Delta = 1$ is a solution to the problem. With $\Delta = 1$ we get $\theta^G = \theta^Z = \theta$, $s^Z = s^G = s$, $z^{uZ} = z^{uG} = z^u$. Moreover we have $v^Z/u^Z = v^G/u^G$, implying that a sector with more vacancies will attract more unemployed searchers. To determine total unemployment, use eq. (4) above. To obtain sectoral employment we need to invoke the demand side, as in eq. (25).

The analysis of tax policies under directed search is analogous to what we have presented in sections 3 and 4. The results are identical.

6.2 Value Added Taxes

Our basic results regarding differentiated payroll taxes carry over to the case with value added taxes (VAT). Let τ^j denote the tax rate on firms' value added and revert to undirected search. The arbitrage equations are then given as follows:

$$rJ_i^j = \frac{P^j(1 - \tau^j)}{P} l_i^j y - \frac{w_i^j l_i^j}{P} + \phi(V^j - J_i^j) \quad (41)$$

$$rV^j = -\frac{P^j \kappa y (1 - \tau^j)}{P} + q(\theta)(J^j - V^j) \quad (42)$$

where P^j is the consumer price and $P^j(1 - \tau^j)$ the producer price. With free entry of vacancies we obtain a free-entry condition for sector j :

$$\frac{w^j l^j}{P^j(1 - \tau^j)} = y \left[l^j - \frac{\kappa(r + \phi)}{q(\theta)} \right] \quad (43)$$

Comparing with eq. (11) above we have

$$(1 - \tau^j) = \frac{1}{1 + t^j} \quad (44)$$

Proceeding to the wage equations we obtain counterparts to (16) and (17) with $(1 + t^j)$ replaced by $(1 - \tau^j)^{-1}$; this holds also in the expression for Δ . We also rediscover (19) and (20). Hence our results regarding the effects of payroll taxes carry over to value added taxes. The welfare analysis is also exactly the same when $r = 0$. Note that the tax base with VAT is:

$$T^{VAT} = \sum_{j=G,Z} P^j (n^j l - v^j \kappa) y = \sum_{j=G,Z} P^j n^j (l - \kappa \phi \theta^\eta) y \quad (45)$$

since $q(\theta)v^j = \phi n^j$. The tax base with payroll taxes is

$$T^{PT} = \sum_{j=G,Z} w n^j l = \sum_{j=G,Z} P^j n^j [l - \kappa(r + \phi)\theta^\eta] y \quad (46)$$

so the two tax bases coincide when $r = 0$. This is simply an implication of the fact that a VAT applies to the wage bill plus aggregate profits, noting that aggregate profits is given as $\Pi = (1 - u)(yl - wl) - v\kappa y = (1 - u)r\kappa y\theta^\eta$. Hence, $\Pi = 0$ as $r = 0$.

6.3 Unemployment Benefits

We have ignored unemployment benefits in the main analysis; indeed, there is no rationale for benefits in this economy with risk neutral agents. For the sake of realism, consider benefits paid to unemployed workers indexed to (general) labor earnings at the fixed replacement rate ρ , i.e., $B^j = \rho w^j l^j$, with $\rho \in [0, 1)$. B^j is taken as fixed in the wage bargains although it is endogenous to the general wage levels. The equations for bargained real wages are then obtained as:

$$\begin{aligned} \frac{w^G (1 + t^G) l^G}{P^G} &= \beta_1 \left[(z^u - z^G) \left(\frac{P^Z}{P^G} \right) (1 + t^G) \right] \\ &\quad + \beta_2 [y (l^G + \kappa s \theta^G) + y \kappa s \theta^Z \Delta^{-1}] \end{aligned} \quad (47)$$

$$\begin{aligned} \frac{w^Z (1 + t^Z) l^Z}{P^Z} &= \beta_1 (z^u - z^Z) (1 + t^Z) \\ &\quad + \beta_2 [y (l^Z + \kappa s \theta^Z) + \beta y \kappa s \theta^G \Delta] \end{aligned} \quad (48)$$

where $\beta_1 = (1 - \beta) / [1 - (1 - \beta)\rho]$ and $\beta_2 = \beta / [1 - (1 - \beta)\rho]$. Note that $\beta_1 + \beta_2 \leq 1$ as $\rho \geq 0$. By invoking the free entry conditions we can derive:

$$\kappa (r + \phi) \theta^\eta = (1 - \beta_2) l - \beta_1 \frac{(z^u - z^e) (1 + t^Z)}{y} - \beta_2 \kappa s \theta \quad (49)$$

where $\Delta = 1$ is imposed. The right-hand side of (49) is invariant to derivative changes in s . However, it is generally *not* invariant to changes in l ; it is straightforward to verify that the right-hand side is decreasing in l for $\rho > 0$. The envelope property that holds for $\rho = 0$ does not carry over to the case with $\rho > 0$, the reason being that workers and firms do not internalize the effects on benefits of their wage decisions.

As is clear from (49), taxes on goods have no effect on tightness; this result is crucially dependent on the assumption that benefits are indexed to wages through a fixed replacement rate. The tax rate on services generally influences tightness, however. In addition to the effect already discussed, there is now also an induced “benefit effect” that operates through work-hours. By differentiation of (49) we obtain:

$$\text{sign} \frac{\partial \theta}{\partial t^Z} = \text{sign} \left[z^e - z^u + \rho \xi_t^l \left(\frac{y l}{1 + t^Z} \right) \right] \quad (50)$$

where $\xi_t^l \equiv -\partial \ln l / \partial \ln(1 + t^Z)$ is the elasticity of work-hours with respect to the tax rate. The inequality $z^e < z^u$ is no longer sufficient to guarantee a negative sign when $\rho > 0$. A cut in the service sector tax rate induces a rise in work-hours and thereby in the benefit level; this in turn tends to raise wage pressure. The net effect on tightness is in general ambiguous.

We have undertaken a number of calibrations with a positive replacement rate ($\rho = 0.3$). With two exceptions, the same parameters as in the earlier

simulations were used. The exceptions pertain to a and m ; we now set $a = 0.3$, and $m = 0.008$ so as to obtain reasonable unemployment figures. The experiments always suggest that tax differentiation increases employment and welfare. The welfare gains from optimal differentiation is non-negligible. For example, the gain amounts to 2 percent of total consumption when the government absorbs 26 percent of GDP.

6.4 Hours Determined by the Worker

Our next variation on the theme briefly considers the case where work-hours are set at the employed worker's discretion. Suppose that the employed worker allocates time so as to maximize rE^j , taking the wage as given. Assuming an interior solution, this yields a familiar "profit maximization" condition:

$$z'(h^j) = \frac{w^j}{PZ} \quad (51)$$

implying that the marginal productivity of home production equals the real wage in units of services. Since the production function is strictly concave, it follows immediately that a rise in the wage causes a reduction in time spent in home production and an increase in time spent in market work: $\partial h^j / \partial w^j < 0$ and $\partial l^j / \partial w^j > 0$. The first-order condition for the Nash bargain can be written as:

$$E^j - U = \left(\frac{\beta}{1 - \beta} \right) \left[1 - \varepsilon^j \left(\frac{P^j y}{w^j(1 + t^j)} - 1 \right) \right]^{-1} \frac{J^j}{1 + t} \quad (52)$$

where the free entry condition $V = 0$ is imposed and $\varepsilon^j \equiv w^j l'(w^j) / l(w^j) > 0$ is the wage elasticity of labor supply. The expression in the squared brackets must be positive for an interior solution of the wage bargain. A higher wage has a direct negative effect of the value of the firm but also an offsetting positive effect arising from the fact that the higher wage encourages labor supply. The cost to the firm of a higher wage is declining in the wage elasticity of labor supply.

One can work out the comparative statics of this problem along the same lines as with bargaining over wages. The results are similar although somewhat more complex. The additional complexity arises because the tax influences work-hours through its effect on the wage. Our numerical exercises suggest that the welfare gains from tax differentiation are of the same order of magnitude as with bargaining over hours.

6.5 Distributional Issues

We have so far ignored distributional issues. Indeed, distributional conflicts do not appear as long as workers are identical and discounting is ignored. In this case workers' "permanent incomes" are identical, i.e., $rU = rE$, and the timing of spells of unemployment and employment does not matter. However, if $r > 0$ timing does matter and $rE > rU$. We have:

$$rU = \left[\frac{(r + \phi) P^Z z^u + \alpha(\theta) [wl + P^Z z^e]}{r + \phi + \alpha(\theta)} + R + \pi \right] P^{-1} \quad (53)$$

$$rE = \left[\frac{\phi P^Z z^u + [r + \alpha(\theta)] [wl + P^Z z^e]}{r + \phi + \alpha(\theta)} + R + \pi \right] P^{-1} \quad (54)$$

We have examined the distributional implications of maximization of steady state welfare, as given by (28), for annual interest rates equal to 0.05 and 0.10, respectively. Table 4 presents results for $r = 0.10$. The optimal policy causes a tiny decline in the permanent income ratio, i.e., rU/rE .¹² However, tax differentiation improves steady state levels of welfare for both employed and unemployed workers. The fact that both groups gain from tax differentiation is driven by a higher level of labor market tightness associated with the optimal policy.

¹²For $\bar{R} = 29$ the ratio is 0.987 with uniform taxation and 0.986 with optimal differentiation.

Table 4. The Effects of Tax Differentiation with Discounting ($r = 0.10/365$) and Cobb-Douglas utility. Parameters (except r) as in Table 1.

\overline{R}	t	t^G	t^Z	u	u	ΔW	rU	rU	rE	rE
	unif	opt	opt	unif	opt	%	unif	opt	unif	opt
10	.14	.16	.11	.064	.064	0.0	90.9	90.9	92.7	92.7
20	.35	.41	.25	.069	.066	0.3	90.4	90.6	91.9	92.2
25	.52	.60	.34	.074	.068	1.0	89.4	90.2	90.7	91.6
26	.57	.64	.35	.076	.069	1.3	89.0	90.1	90.3	91.4
27	.62	.69	.37	.079	.069	1.7	88.5	89.9	88.5	91.3
28	.69	.74	.39	.083	.070	2.4	87.7	89.8	88.9	91.1
29	.82	.80	.41	.092	.070	4.2	86.1	89.6	87.2	90.9

7 Concluding Remarks

We have developed a two-sector general equilibrium model of search unemployment in order to examine the case for sectoral tax differentiation. In particular, we have analyzed how taxes affect labor market outcomes when services produced in the market can also be produced within the household. The analytical results are unambiguous when unemployment benefits are ignored: a tax cut on services reduces unemployment whereas a tax cut on goods has no effect. A reform that introduces tax differentiation, with lower taxes on services, is also welfare improving. The numerical results suggest that the welfare gains from optimal tax differentiation may well be substantial; this holds irrespective of whether or not unemployment benefits are taken into account. Of course, the specific numbers are sensitive to the details of the calibration but the general features of the results appear to be fairly robust.

All workers are *ex ante* identical in our analysis; heterogeneity arises *ex post* as some workers become employed whereas others become unemployed. This is a useful simplification as long as we focus on efficiency aspects of sectoral tax differentiation. It is however an unsatisfactory assumption if

one takes distributional issues seriously. Indeed, one argument sometimes voiced in favor of lower taxes on household services is that such reforms might encourage employment especially among less skilled workers. To address this issue, the analysis has to be extended to incorporate heterogeneous labor.

Finally, we suggest that our framework can be usefully adapted to an analysis of tax evasion behavior and policies to prevent tax evasion.¹³ This, however, would be a different paper.

Appendix A: Proof of Lemma 1

We want to show that $h^u > h^j$, and hence $s < l^j$ and $z^u > z^j$ for $j = G, Z$. First, use eqs. (8), (10) and (12) to obtain:

$$z'(h^u) = (\Delta\theta^G + \theta^Z) \left(\frac{\beta}{1-\beta} \right) \left(\frac{\kappa y}{1+t^Z} \right) \quad (\text{A1})$$

Next, note that eqs. (19) and (20) in the main text can be written as:

$$\theta^G \Delta + \theta^Z = \frac{(1-\beta)}{\beta \kappa s} \left[l^G \Delta - \frac{1}{y} (z^u - z^G) (1+t^Z) \right] - \frac{\Delta (r+\phi) \theta^\eta}{\beta s} \quad (\text{A2})$$

$$\theta^G \Delta + \theta^Z = \frac{(1-\beta)}{\beta \kappa s} \left[l^Z - \frac{1}{y} (z^u - z^Z) (1+t^Z) \right] - \frac{(r+\phi) \theta^\eta}{\beta s} \quad (\text{A3})$$

where (A2) corresponds to (19) and (A3) to (20). By inspection of (A1), (A2) and (A3) it is clear that the following inequalities hold:

$$z'(h^u) < \frac{1}{s} \left[l^G + \frac{1}{y} \frac{(z^G - z^u) (1+t^Z)}{\Delta} \right] \frac{y \Delta}{1+t^Z} \quad (\text{A4})$$

$$z'(h^u) < \frac{1}{s} \left[l^Z + \frac{1}{y} (z^Z - z^u) (1+t^Z) \right] \frac{y}{1+t^Z} \quad (\text{A5})$$

The proof is by contradiction. Consider first h^u and h^G and assume $h^u \leq h^G$, i.e., $s \geq l^G$. Since $z(\cdot)$ is strictly concave we have

$$z'(h^u) \geq \frac{z(h^j) - z(h^u)}{h^j - h^u} = \frac{z(h^j) - z(h^u)}{s - l^j} \quad (\text{A6})$$

¹³See Kolm and Larsen (2001) for an analysis along these lines.

for $h^j \geq h^u$, $j = G, Z$. Use (A4) and (A6) to obtain:

$$sz'(h^u) - \frac{l^G y \Delta}{1 + t^Z} < (z^G - z^u) \leq (s - l^G) z'(h^u)$$

implying

$$z'(h^u) < \frac{y \Delta}{1 + t^Z} = z'(h^G)$$

which is a contradiction since $z'(h^u) < z'(h^G)$ implies $h^u > h^G$ and $s < l^G$. Thus $h^u > h^G$, $s < l^G$ and $z^u > z^G$ must hold.

Consider now h^u and h^Z and assume $h^u \leq h^Z$, i.e., $s \geq l^Z$. Use (A5) and (A6) to obtain:

$$sz'(h^u) - \frac{l^Z y}{1 + t^Z} < (z^Z - z^u) \leq (s - l^Z) z'(h^u)$$

implying

$$z'(h^u) < \frac{y}{1 + t^Z} = z'(h^Z)$$

which is a contradiction since $z'(h^u) < z'(h^Z)$ implies $h^u > h^Z$ and $s < l^Z$. Thus also $h^u > h^Z$, $s < l^Z$ and $z^u > z^Z$ must hold. This completes the proof. ■

Appendix B: Proof of Lemma 2

We want to show that $\Delta = 1$ is the unique solution to eq. (21) in the main text. To that end we make the following definitions:

$$\begin{aligned} f(\Delta) &\equiv (1 - \beta) \left[l^Z - \frac{1}{y} (z^u - z^Z) (1 + t^Z) \right] \\ h(\Delta) &\equiv (1 - \beta) \left[l^G - \frac{1}{y \Delta} (z^u - z^G) (1 + t^Z) \right] \\ g(\Delta) &\equiv \kappa (r + \phi) [\theta(\Delta)]^\eta \end{aligned}$$

By invoking these definitions we can rearrange (21) to:

$$\Delta - 1 = \frac{f(\Delta) - h(\Delta)}{h(\Delta) - g(\Delta)}$$

where

$$\text{sign} \left[\frac{f(\Delta) - h(\Delta)}{h(\Delta) - g(\Delta)} \right] = \text{sign} [f(\Delta) - h(\Delta)]$$

since $h(\Delta) - g(\Delta) > 0$ follows from eq. (19) in the main text.

The proof is by contradiction in two steps. We first consider possible solutions involving $\Delta > 1$ and then turn to solutions where $\Delta < 1$.

(i) Assume $\Delta > 1$, i.e., $(\Delta - 1) > 0$

Consider the difference $f(\Delta) - h(\Delta)$ and rearrange to obtain:

$$\begin{aligned} f(\Delta) - h(\Delta) &= \underbrace{(1 - \beta) \left(l^Z - l^G - \frac{(1 + t^Z)}{y\Delta} (z^G - z^Z) \right)}_{\equiv A(\Delta)} - \\ &\quad \underbrace{(1 - \beta) (z^u - z^Z) \frac{(1 + t^Z)}{y} \left(1 - \frac{1}{\Delta} \right)}_{\equiv B(\Delta)} \end{aligned}$$

We have $A(1) = 0$ and also:

$$\frac{dA(\Delta)}{d\Delta} = (1 - \beta) \frac{(1 + t^Z)}{y\Delta^2} (z^G - z^Z)$$

when using eq. (14). Note also that $dz^G/d\Delta = z'(h^G) (\partial h^G/\partial \Delta) < 0$ and thus:

$$\left(\frac{dA(\Delta)}{d\Delta} \right)_{\Delta > 1} < 0$$

so we have $A(\Delta > 1) < 0$.

Since $z^u > z^Z$ by Lemma 1 we obtain $B(1) = 0$ and $B(\Delta > 1) > 0$. We then get

$$\text{sign} [f(\Delta) - h(\Delta)] = \text{sign} [A - B] < 0$$

for $\Delta > 1$. The assumption $\Delta > 1$ thus leads to a contradiction.

(ii) Assume $\Delta < 1$, i.e., $(\Delta - 1) < 0$

Rearrange the expression for $f(\Delta) - h(\Delta)$ as follows:

$$\begin{aligned}
f(\Delta) - h(\Delta) &= \underbrace{(1 - \beta) \left[l^Z - l^G - \frac{(1 + t^Z)}{y} (z^G - z^Z) \right]}_{\equiv C(\Delta)} - \\
&\quad \underbrace{(1 - \beta) \frac{(1 + t^Z)}{y} (z^u - z^G) \left(1 - \frac{1}{\Delta} \right)}_{\equiv D(\Delta)}
\end{aligned}$$

where $C(1) = 0$. We also have:

$$\frac{dC(\Delta)}{d\Delta} = (1 - \beta) \frac{\partial h^G}{\partial \Delta} (1 - \Delta)$$

where $\partial h^G / \partial \Delta < 0$. Thus:

$$\left(\frac{dC(\Delta)}{d\Delta} \right)_{\Delta < 1} < 0$$

implying $C(\Delta < 1) > 0$.

Since $z^u > z^G$ by Lemma 1 we obtain $D(1) = 0$ and $D(\Delta < 1) < 0$ and hence:

$$\text{sign} [f(\Delta) - h(\Delta)] = \text{sign} [C - D] > 0$$

for $\Delta < 1$. The assumption $\Delta < 1$ thus also leads to a contradiction. This completes the proof. ■

Appendix C: Proof of Proposition 2

We want to show that $dW/dt^Z < 0$ when we evaluate at $t^Z = t^G = t > 0$ and recognize $t^G = t^G(t^Z)$ from the government's budget restriction. We have:

$$\left(\frac{dW}{dt^Z} \right)_{t^G(t^Z)} = \frac{\partial W}{\partial t^Z} + \frac{\partial W}{\partial t^G} \frac{\partial t^G}{\partial t^Z}$$

Without loss of generality we set $P^Z = 1$ and consider the derivative:

$$\frac{\partial W}{\partial t^j} = \left(P^G \frac{\partial Y^G}{\partial t^j} + \frac{\partial Y^Z}{\partial t^j} + \frac{\partial P^G}{\partial t^j} Y^G \right) \frac{1}{P} - (P^G Y^G + Y^Z) \frac{\partial P}{\partial P^G} \frac{\partial P^G}{\partial t^j} \frac{1}{P^2} \quad (\text{C1})$$

for $j = G, Z$. To simplify (C1) we need to look at the individual's consumption decision. Suppose that consumer i maximizes a linearly homogenous utility function $v(G_i, Z_i)$ subject to the budget restriction $I_i = P^G G_i + Z_i$. The indirect utility function corresponding to this problem is given as $\tilde{v}_i = I_i/P$, where $P = P(P^G, 1)$ is the price index. Use Roy's identity to derive the demand function as:

$$G_i = -\frac{\partial \tilde{v}/\partial P^G}{\partial \tilde{v}/\partial I_i} = \frac{I_i}{P} \frac{\partial P}{\partial P^G} \quad (\text{C2})$$

and aggregate over all individuals to obtain

$$\frac{G}{I} = \frac{1}{P} \frac{\partial P}{\partial P^G} \quad (\text{C3})$$

where I is aggregate income. Equalize demand and supply in (C3) to obtain:

$$\frac{\partial P}{\partial P^G} = \frac{PY^G}{P^G Y^G + Y^Z} \quad (\text{C4})$$

and by using (C4) to substitute out $\partial P/\partial P^G$ in (C1) we get

$$\frac{\partial W}{\partial t^j} = \left(P^G \frac{\partial Y^G}{\partial t^j} + \frac{\partial Y^Z}{\partial t^j} \right) \frac{1}{P}$$

which implies

$$\text{sign} \frac{\partial W}{\partial t^j} = \text{sign} \frac{\partial (Y^G + Y^Z)}{\partial t^j}$$

when evaluated at $t^G = t^Z = t$.

Consider next:

$$Y^G + Y^Z = (1 - u) (l - \kappa \phi \theta^\eta) y + (1 - u) z^e + u z^u$$

and recall that Proposition 1 implies $\partial (Y^G + Y^Z) / \partial t^G = 0$. Hence $\partial W / \partial t^G = 0$ when evaluating at $t^G = t^Z = t$. We can thus conclude that:

$$\left(\frac{dW}{dt^Z} \right)_{t^G(t^Z)} = \frac{\partial W}{\partial t^Z}$$

when $t^G = t^Z = t$. We can then proceed by deriving $\partial (Y^G + Y^Z) / \partial t^Z$. After some tedious algebra we obtain:

$$\frac{\partial (Y^G + Y^Z)}{\partial t^Z} = \Omega_1 (\beta - \eta) \frac{\partial \theta}{\partial t^Z} + t \left[\Omega_2 \frac{\partial \theta}{\partial t^Z} + \Omega_3 \frac{\partial s}{\partial t^Z} + \Omega_4 \frac{\partial l}{\partial t^Z} \right]$$

where

$$\begin{aligned} \Omega_1 &\equiv \frac{y\kappa su}{1 - \beta} > 0 \\ \Omega_2 &\equiv \frac{(1 - \eta)}{\theta} (1 - u) (z^u - z^e) u > 0 \\ \Omega_3 &\equiv \frac{\beta}{(1 - \beta)} \kappa \theta u \frac{y}{1 + t} - (z^u - z^e) \frac{\partial u}{\partial s} > 0 \\ \Omega_4 &\equiv (1 - u) \frac{y}{1 + t} > 0 \end{aligned}$$

Moreover, $\partial \theta / \partial t^Z < 0$, $\partial s / \partial t^Z < 0$ and $\partial l / \partial t^Z < 0$. We thus have: (i) $dW/dt^Z = 0$ if $\beta = \eta$ and $t = 0$; (ii) $dW/dt^Z < 0$ if $\beta \geq \eta$ and $t > 0$. ■

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